

It can be seen from eqns. 25-27 that the matrices  $P_{11}$ ,  $P_{21}$ , and  $P_{22}$  needed to compute the control law (eqn. 20), can be expressed in terms of  $P_{22}^0$ ,  $\psi_0$ ,  $C_0$  and the given quantities  $F$  and  $Z$ . This would prove useful to establish the applicability of efficient procedures such as the integration-free and partitioned Riccati algorithms to the fixed end-point problem. This is because the solution of the Riccati equation via these fast algorithms requires calculation of  $\psi_0$ ,  $C_0$ , and  $P_{22}^0$  over each interval, and all that is subsequently needed to determine the control law or minimum performance index for this problem, is to use eqns. 25-27 to obtain  $P_{22}$ ,  $P_{11}$ , and  $P_{21}$ . The increase in computational complexity is minimal.

We conclude by noting one advantage of the procedure described above is that the terminal constraints enter the problem at the last stage. Thus it becomes economical when it is required, to solve the problem for various sets of terminal constraints. Also, it allows one to take advantage of the computational benefits that result for special choices of terminal conditions in certain fast algorithms.<sup>4</sup> The doubling algorithm proves especially useful when studying the limiting behaviour of the system, while the partitioned Riccati algorithm allows the advantages of parallel computation of the solution together with improved numerical accuracy through the special choice of terminal conditions. Further details in this regard may be found in References 1 and 4. From the new formulation presented here we see that the computational procedures along with their numerical benefits can be extended to the fixed end-point problem as well.

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#### References

- BIERMAN, G. J., and SIDHU, G. S.: 'Integration-free interval doubling for Riccati equation solutions', *IEEE Trans.*, 1977, AC-22, pp. 831-834
- BRYSON, A. E., and HO, Y. C.: 'Applied optimal control' (Blaisdell, Waltham MA, 1969)
- BROCKETT, R.: 'Finite dimensional linear systems' (Wiley, New York, 1969)
- LAINIOTIS, D. G.: 'Partitioned Riccati solutions and integration-free doubling algorithms', *IEEE Trans.*, 1976, AC-21, pp. 677-689
- WARRIOR, J., and VISWANADHAM, N.: 'Scattering theory and linear optimal control: Regulator and servo problems' (submitted for publication)
- ZACHRISSON, L.: 'On optimal smoothing of continuous-time Kalman processes', *Inf. Sciences*, 1969, 1, pp. 143-172

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## 100 K UNCOOLED GaAs M.E.S.F.E.T. AMPLIFIER AS PARAMP REPLACEMENT

Indexing term: Microwave parametric amplifiers

The characterisation, design and realisation of a low noise GaAs m.e.s.f.e.t. amplifier to replace a narrow band parametric amplifier at 1.7 GHz is described. Special measurement and analytical techniques are necessary owing to the high reflection coefficients and conditional stability of the transistor, as well as the very low noise figures being measured. A single stage amplifier is realised with a noise figure of 1.25 dB and 13.5 dB associated gain at 1.7 GHz.

**Introduction:** Low noise uncooled paramps are widely used in Earth receiving satellite stations.<sup>1</sup> In order to obtain a low overall noise figure it is often necessary to operate these amplifiers under high gain (14 dB) conditions thereby limiting their useful bandwidth to less than 60 MHz. This poses serious problems with regard to setting up at the desired frequency,

and stabilisation against gain/frequency drift. GaAs m.e.s.f.e.t. amplifiers offer the advantages of a much wider bandwidth (virtually independent of gain requirements), high reliability, compactness and an economic price for similar noise temperature performance. However the high terminal reflection coefficients and the conditional stability of the GaAs m.e.s.f.e.t. below 4 GHz, as well as the very low noise figures being measured necessitate the use of special characterisation and design procedures in order to enable the transistor to be exploited to full advantage. This letter describes the measurement and circuit design techniques which led to the realisation of a 1.25 dB noise figure, 13.5 dB gain amplifier at 1.7 GHz.

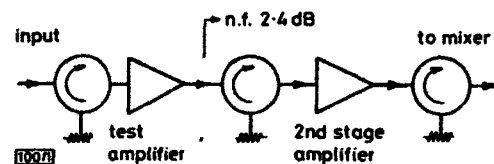


Fig. 1

**Device characterisation:** An AILTECH solid-state noise generator, calibrated against an NBS standard hot and cold noise source, was used with an RHG double balanced mixer followed by an AIL receiver and noise figure indicator (PANFI type 75), in the noise measurement procedure.

It was necessary to improve the measurement accuracy of the noise bench by reducing the masking effect of the large correction factor due to the 5 dB mixer noise figure. An elegant manner of accomplishing this is to use a 2 stage GaAs m.e.s.f.e.t. test unit, with the second stage matched to have a low overall noise figure (2.4 dB in our case). By inserting an isolator between the 2 stages, the optimum-noise-figure/noise-impedance of a transistor and its associated gain could be accurately measured in the first stage of the test unit with only a minor correction (typically 0.2 dB) to the noise figure (Fig. 1).

At 1.7 GHz, 1  $\mu$ m gate GaAs m.e.s.f.e.t.s have high input and output reflection coefficients (e.g.  $S_{11} = 0.95 \angle -65^\circ$ ,  $S_{22} = 0.8 \angle -34^\circ$ ) and are only conditionally stable. In order to reduce the high losses associated with tuning out reflection coefficients of this magnitude, it is necessary to match as close to the transistor as possible. If, in addition, the tuning is carried out in the same medium as the amplifier is to be realised, the tuning losses and matching conditions should be similar to those that will be encountered in the final amplifier. Using a teflon fibreglass substrate ( $\epsilon_r = 2.65$ ,  $h = 0.8$  mm) with 50  $\Omega$  microstrip lines, approximately  $\lambda/2$  at 1.7 GHz, at the input and output ports, the tuning was carried out using 14 mm diameter discs. A resistance was incorporated into the output bias filter in order to absorb low frequency gain and stabilise the transistor. By this means a narrow band 'amplifier' was obtained, centred at 1.7 GHz, which gave an optimum noise figure of 1.4 dB and associated gain of 14 dB, uncorrected for isolator and circuit matching losses, for an NEC 244083.

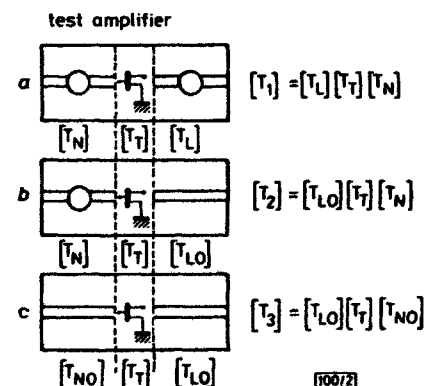


Fig. 2

To obtain the noise source impedance and the load impedance necessary to achieve the performance measured on the test amplifier, an improved version of the 'peeling' method of Soares and Cripps<sup>2</sup> was used. This was effected by carrying out a set of *S*-parameter measurements at 3 frequency points:

- (a) the *S*-parameters of the test 'amplifier' were obtained
- (b) the output matching discs were removed and the test 'amplifier' was remeasured
- (c) the input matching discs were removed and the *S*-parameters of the GaAs m.e.s.f.e.t. were obtained.

From these 3 measurements the load impedance  $\Gamma_L$ , and the optimum noise source impedance  $\Gamma_n$ , could be obtained by a series of inverse *T*-matrix multiplication procedures (see Fig. 2) using an inhouse computer program LMANAL 3.

The above procedures were carried out for test 'amplifier' centred at 1.7 GHz, and the noise source and optimum load impedances were obtained for 1.6, 1.7 and 1.8 GHz. They are given in Table 1.

Table 1

Frequency GHz	Noise source impedance	Load impedance
1.6	0.93/52.4°	0.72/78.5°
1.7	0.81/58.1°	0.70/67.5°
1.8	0.89/49.8°	0.78/58.5°

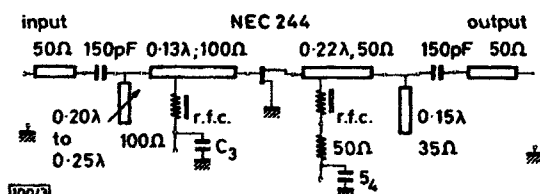


Fig. 3

**Amplifier:** Once the noise source and load impedances were obtained, input and output circuits were synthesised to provide these impedances from 50Ω terminations. Two particular features of the circuits chosen (Fig. 3) were: the use of the open-circuit stub in the input circuit to compensate for

variations in transistor *S*-parameter spread, and the use of a resistive element in the output circuit to absorb out-of-band gain. As the noise match impedance ( $0.8 < +58^\circ$ ) differs substantially from the input conjugate match impedance ( $0.95 < +65^\circ$ ), isolators were incorporated at the input and output ports and the performance of the ensemble was measured. The design details of the microstrip amplifier, which was realised on teflon fibreglass, are given in Fig. 3.

**Experimental results:** As we estimate our experimental error to be in the order of  $\pm 0.15$  dB, the device noise figure of  $(1.05 \pm 0.15)$  dB at 1.7 GHz for the NEC 244083 measured compares well with the 1.15 dB noise figure predicted at 2 GHz by the Van der Ziel/Baechtold model.<sup>4</sup>

The overall amplifier, including EMEL garnet isolators centred at 1.695 MHz with an insertion loss of less than 0.15 dB provided a noise figure of 1.25 dB with gain of 13.5 dB at 1.7 GHz, a performance comparable to that achieved by commercial uncooled paramps.

However no paramp can match the performance of the m.e.s.f.e.t. over the bandwidth 1.6–1.8 GHz: maximum noise figure 1.6 dB with  $13.5 \pm 0.2$  dB gain.

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#### References

- 1 FREEMAN, R. L.: 'Telecommunication transmission handbook' (Wiley-Interscience, 1975)
- 2 SOARES, R. A., and CRIPPS, S. C.: 'Some novel characterization techniques used in the design of X-band GaAs FET amplifiers', 1976 European Conference Proceedings, pp. 19–23
- 3 SOARES, R. A.: 'Microwave amplifier design using bipolar transistors', Ph.D. thesis, London University, October 1974
- 4 BAECHTOLD, M.: 'Noise behaviour of Schottky barrier gate field effect transistors at microwave frequencies', *IEEE Trans.*, 1971, ED-18, pp. 97–104

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## GOOS-HÄNCHEN SHIFT FOR LEAKY RAYS ON STEP-INDEX WAVEGUIDES

Indexing terms: Optical fibres, Optical waveguide theory

A universal expression is presented for the lateral shift of leaky rays incident at angles close to the critical angle on a curved interface between uniform dielectric media of slightly different refractive indices.

**Introduction:** When a bound ray propagates along a lossless step-index waveguide of arbitrary cross-section, it is confined to the core by successive reflections from the core-cladding interface. The classical laws of Snell determine the direction of the ray trajectory after each reflection, and the power propagating along the ray path is conserved by Fresnel's laws.

Associated with the bound ray in the core, there is an evanescent electromagnetic field in the cladding which extends from the core-cladding interface, and gives rise to the Goos-Hänchen, or lateral shift. This shift translates the origin of the reflected ray a distance *S* along the interface in the plane defined by the incident ray and normal, as illustrated in Fig. 1. The incident and reflected rays are the local plane wave components of the fields in the core. On reflection there is a change in phase  $\phi$  between these two waves, and the extent of the shift *S*, for both planar<sup>1</sup> and curved interfaces,<sup>2</sup> can be

expressed as

$$S = -\frac{1}{n_{co}k \cos \alpha_i} \frac{d\phi}{d\alpha_i} \quad (1)$$

where  $\alpha_i$  is the angle of incidence relative to the normal,  $n_{co}$  is the core refractive index, and  $k = 2\pi/\lambda$ , with  $\lambda$  being the

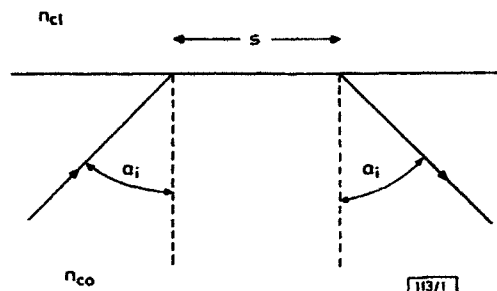


Fig. 1 Sketch illustrating the shift of the origin of a reflected ray along a dielectric interface in the plane of the incident ray and the normal to the interface

The angles of incidence and reflection are  $\alpha_i$  measured with respect to the normal to the interface